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Vibration Study of Delaminated Carbon Fibre Reinforced Polymer Composite Plate for Clamped-clamped Boundary Conditions

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Abstract. The vibration of the delaminated composites concerns the structure safety and dynamic behaviour of the composite structures as it can be vital in the presence of delamination. In this research paper, analytical analysis, experimental work and finite element simulations are combined to analyze the vibration behaviour at different delamination size, different stacking sequences. The boundary condition in this investigation was all sides clamped. Analytical results were obtained using the first order shear deformation theory. Rayleigh-ritz method was used to derive the governing equations to find the natural frequencies and the results were computed using Mat lab tool. Experiments have been conducted to study the vibration characteristics of carbon fiber reinforced polymer (CFRP) composite plate. The finite element analysis was done using software package, ANSYS is used to fetch the vibration response of carbon fibre reinforced polymer composite plate for boundary conditions, stacking sequences and delamination sizes. The results from analytical, experimental and finite element analysis were then compared and verified that the maximum percentage of error is ignorable. It is seen that the natural frequencies of carbon fibre reinforced polymer decreased with an increase in delamination size. Stacking sequence of (0/90/45/90) showed higher values of natural frequencies subjected to allsided clamped boundary conditions. It was interesting to know that there were small differences in values of natural frequencies for lower modes but the difference gradually increased in case of higher modes.

Keywords: finite element analysis, composites, delamination, experimental vibration

Introduction

The application of composites in various fields of sporting equipment, aerospace, marine and agricultural products have increased tremendously due to their multi-dimensional, attractive and novel properties (Aksencer and Aydogdu, 2018; Agarwal *et al*., 2017; Jiang *et al*., 2017; Kamar *et al*., 2017; Kumar *et al*., 2017; Saghafi *et al*., 2017; Yelve *et al*., 2017; Kharghani and Guedes Soares, 2016; Imran, 2015) along with low maintenance of composite materials (Asmatulu *et al*., 2018; Imran *et al*., 2018; Saghafi *et al*., 2017; Yelve *et al*., 2017; Kharghani and Guedes, 2016; Bakis *et al*., 2002).

One of the most critical defect in composites is delamination that can badly affect the behaviour of composite structures (Imran *et al*., 2018; Venkate *et al*., 2017) so it is equally important to study the effect of ply layups, boundary conditions and delaminated region on the vibration characteristics of composite structures. Delamination is comparatively most complex problem that involves material and geometry dis-continuations (Imran *et al*.,2018).

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It is most important to have the vibration characteristics of structures investigated prior to be applied in order to improve the design parameters (Shao *et al*., 2017). A considerable studies, on the behavior of composite structures like beams, shells and plates are available however, studies on the influence of stacking sequences and delamination size parameters on the delaminated composite plates are scarce. In following paragraphs, we will do aerial exploration of the work already carried out on the composite structures subjected to delamination and without delamination along with the methodologies used to analyze the effect on the vibration characteristics of different structures (Imran *et al*., 2019 a and b).

The behaviour of delaminated composites under vibration has been extensively studied analytically. It was found that delamination size and location badly affects the vibration behaviour of the composite structures (Jadhav and Bhoomkar, 2016; Shu and Della, 2004; Brandinelli and Massabò, 2003; Kim and Hwang, 2002; Lee *et al*., 2002). Experimental results showed that fundamental frequency further decreased in the presence of matrix. However, this decrease in fundamental frequency was not significant for small delamination (Chawla and Ray-Chaudhuri, 2017; Lee *et al*., 2003; Thornburgh and Chattopadhyay, 2003; 2001; Luo and Hanagud, 2000; Luo *et al*., 1997). It was concluded that the values of natural frequencies for clamped square plate with delamination decreased significantly with increase in delamination size.

Finite element analysis is an effective tool for the prediction of the structural behaviour under loadings like static, dynamic, thermal and vibration. The behaviour of delaminated composites under vibration has been extensively studied using commercial software packages like ANSYS and ABAQUS (Imran *et al*., 2019 a and b; Yurddaskal *et al*., 2018; Zhang *et al*., 2018; Juhász *et al*., 2017; Mallik and Rao, 2017; Vo *et al*., 2017; Yashavantha, 2017; Hirwani *et al*., 2016; Shukla and Harsha, 2016; Sadeghpour *et al*., 2016; Zhu *et al*., 2012).

From the literature, it is observed that the vibration analysis on the carbon fibre reinforced polymer composite plates subjected to CCCC boundary condition for (0/90/45/90), (0/45) and (0/90) is very limited and the availability of the vibration behaviour for this specific structure is poor (Oliveri and Milazzo, 2018; Vescovini *et al*., 2018; Ardestani *et al*., 2017; Sayyad and Ghugal, 2017).

The critical applications of composite in aeroplane wings, bridges and columns are mostly used CCCC. Therefore it is utmost important to investigate the vibration characteristics of CFRP composite plate under these constraints.

In this paper, the vibration investigation of delaminated and non-delaminated under CCCC constraint is performed using analytical, experimental and finite element analysis techniques. To study the effect of delamination size on the vibration properties of CFRP composite plate, delamination of 6.25%, 25% and 56.25% of the total plate area were incorporated at the middle of the rectangular plate. Three stacking sequences (0/45/90/45), (0/90) and (0/45) are investigated experimentally and FEA with the above delamination sizes.

The following section provides the detailed investigations which is followed by results and discussion.

Analytical analysis. It is found in the first-order displacement theory as follows. An 8-nodded plate or sheet element & 5-degree of self-determination at every node is obtainable here for finite element model.

Nomenclature. A, B, C and D are coefficients of frequency; a, b and ρ are width, height and density of the plate respectively; Dx, Dy, Dxy are the stiffness terms and depends on the orientation of the fibres; E_x=Young's Modulus-Longitudinal; E_y= Young's Modulus – Transverse; m, n= Eigen vectors for mode shapes and mode numbers; ω= Angular velocity; f_1: first natural frequency; G_{xy}= Shear modulus - in-plane; V_{xy}= Poisson′s ratio

$$u=\sum_{i=1}^{8} u_i N_i \quad v=\sum_{i=1}^{8} u_i N_i \quad v=\sum_{i=1}^{8} u_i N_i \quad w=\sum_{i=1}^{8} w_i N_i \quad \theta_x=\sum_{i=1}^{8} \theta_{xi} N_i \quad \theta_y=\sum_{i=1}^{8} \theta y_i N_i \quad \text{........................} \quad (1)$$

where;

Ni represents “shape function” and ‘i’ represents “node number”.

Strain-stress relationship in global co-ordinate axes scheme or system has taken the following form from standard books.

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_x \\ M_{xy} \end{bmatrix} = \begin{bmatrix} a & b \\ b & d \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ Y_{xy} \\ K_x \\ K_x \\ K_{xy} \end{bmatrix} \quad \text{.......................................} \quad (2)$$

Extensional stiffness matrix, extensions-bending coupling matrix and bending stiffness matrix can be extracted from the standard notations and can be rewritten as follows;

$$[a_{ij}]=\sum_{K=1}^{N} [Q_{ij}]_k \ (Z_k - Z_{k-1})$$

$$[b_{ij}]=\frac{1}{2}\sum_{K=1}^{N} [Q_{ij\ k} - (Z^3_k - Z^3_{k-1})$$

$$[d_{ij}]=\frac{1}{2}\sum_{K=1}^{N} [Q_{ij}]_k \ (Z_k - Z_{k-1}) \quad \text{.............................} \quad (3)$$

$[a_{ij}]$ = ‘extensional stiffness matrix’
$[b_{ij}]$ = ‘extension-bending coupling matrix’
$[d_{ij}]$ = ‘bending stiffness matrix’
‘Lamina Stiffness Matrix’ is presented as,

$$[D]=\begin{bmatrix} a & b \\ b & d \end{bmatrix}_k \quad \text{..} \quad (4)$$

Strain-stress relation for ‘shear forces’ is given by,

$$\begin{bmatrix} Q_x \\ Q_Y \end{bmatrix} = \sum_{k=1}^{N} K1 {}^{Z_K}_{Z_K} \begin{bmatrix} Q_{55} & Q_{45} \\ Q_{55} & Q_{44} \end{bmatrix} \begin{Bmatrix} Yxz \\ Yyz \end{Bmatrix} \; dz \; .... (5)$$

where;

N = layer number in coat or laminate and, k = the specific layer
K1 represents ‘shear correction factor’.
a, b and d matrix is obtained

The ‘eigen value’ shape of governing eq. for finding ‘natural frequency’ of given scheme or system is expressed as;

$$([K]-\omega^2 [M])\Delta = 0 \; .... (6)$$

Experimental setup and procedure. The composite plates models used in the present work were made of carbon fibers. 0° and 90° direct roving were used for interweaving the fabrics with epoxy matrix. Ratio of 50:50 in weight of the fibre and matrix was used for sample preparation. The individual materials used for preparation of samples were epoxy as resin, hardener as catalyst, carbon fibres rovings as reinforcement and polyvinyl alcohol as releasing agent. Samples were prepared using handy layup method and cured at room temperature. During fabrication, Teflon tape was used to generate artificial delamination of sizes 6.25%, 25% and 56.25% of the area of the rectangular plate. These delamination were incorporated at the mid plane of the plate after each four layers as total layers were eight. All the models were subjected to free vibrations. The natural frequencies for first twelve modes of eight-layered carbon fiber reinforced polymer composite plate were determined experimentally with and without delamination. The experiments were performed using the analyzers, transducers and modal hammer as shown in Fig. 1.

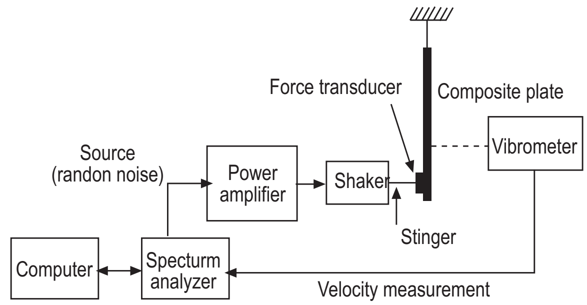

Fig. 1. Simple layout of experimental setup.

The LABVIEW software was used during the vibration measurements. The plates were exited using impact hammer on the random points and the required data was received through data acquisition component. Signals obtained from data acquisitions system were then processed in LABVIEW software. The acceleration signals were then processed through power spectrum module to fetch the natural frequencies along with their mode shapes.

Finite element approach using ansys. Figure 2 shows the schematic diagram of delaminated composite plate with mid plane delamination. Delaminated region was modeled by merging the nodes in contact while other nodes of the area were keep unmerged considering the non-delaminated area. This methodology is extensively adapted to model the delamination in composite structures. Cohesive zone model or virtual crack closure techniques were not used because the proposed delamination were not bonded. Triangular elements were used to mesh the plate for the vibration properties of delaminated composite plate. All the modeling, meshing and simulation were done in ANSYS APDL version 17 software package. For the simulations solid elements were used in the thickness directions for each prepeg.

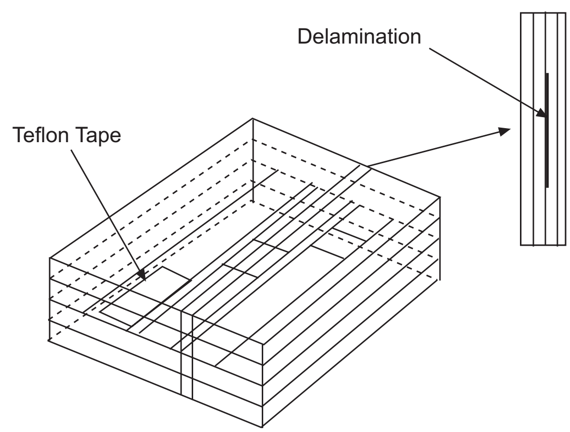

Fig. 2. Schematic view of delamination (Lee *et al*., 2003)

Results and Discussion

Results from analytical analysis, experimental analysis and finite element are presented for free vibration of composite plate with mid-plane delamination as shown in Table 1.

Based on the above three analysis techniques of analytical analysis, experimental analysis and finite element methods; Results obtained are validated on comparison for free vibration of laminated composite plate. By careful observation on the results obtained, it reveals that the results from all three techniques are in very good agreement.

In the present investigation, experimental, numerical and finite element study are carried out for an eight-layered (0/90/45/90)2, (0/45)4 and (0/90)4 carbon fibre reinforced polymer composite plate. The geometrical dimensions considered for carbon fibre reinforced polymer composites are 0.15m of each length and width of the plate and 0.02m is taken as thickness of the plate. Square size delamination was incorporated at the mid plane of the plate. In the present study, effect of delamination size, boundary conditions and stacking sequences on the natural frequencies are studied.

Effect of delamination size. In this section, we will find the influence of delamination size on the natural frequencies of CFRP composite plate with mid plane delamination.

The Table 2 reveals that the on increase in delamination size reduces the natural frequencies of carbon fibre reinforced composite plate for (0/90/45/90), (0/45) and (0/90) for C-C-C-C boundary conditions. It has been observed that there is negligible effect of delamination in the lowest mode and its effect significantly increased for higher modes. The behaviour is similar in all three techniques. The natural frequency for delamination size of 56.25% has lowest value rather than 6.25% delaminated size where the value is higher. It is observed that for all delamination sizes, the natural frequencies are less affected for lower modes as compared to the higher modes because there is less difference in delamination between 6.25% and 25% as compared to 6.25% with 56.25%. It is also due to that at higher delamination sizes, bonding between the laminates

Table 1. Effect of all sides clamped and all sides simply supported constraints and effect of stacking sequence for first four modes

Boundary condition	Layers sequence	Method	Mode1 , Hz	Mode2 , Hz	Mode3 , Hz	Mode4 , Hz
All sides clamped		Experimental	872	1781	1801	2508
	0/90/45/90	FEA	879.65	1795.6	1795.6	2547.6
		Analytical	886.79	1822.9	1822.9	2522.2
		Experimental	871	1765	1781	2649
	0/45	FEA	873.84	1773.3	1773.3	2602.5
		Analytical	882.411	1802	1802	2659.03
		Experimental	878	1828	1831	2499
	0/90	FEA	884.65	1815.7	1815.7	2487.6
		Analytical	886.79	1837.03	1837.03	2522.2

Table 2. Effect of delamination size for C-C-C-C boundary condition of (0/90/45/90), (0/45) and (0/90) stacking sequences

Delamination area	Stacking sequence	1		2		3		4	
		Exp	FEA	Exp	FEA	Exp	FEA	Expl	FEA
6.25%	0/90/45/90	883.41	889.36	1803.8	889.36	1803.8	1889.36	2538	2601
25%		883.41	880.68	1801.7	1803.7	1801.7	1835	2538	2567.1
56.25%		883.38	877.85	1795.2	1801.1	1795.2	1819.8	2537.4	2527.8
6.25%	0/45	876.31	883.93	1776.4	1799.4	1776.4	1804.2	2601.4	2657.3
25%		876.31	874.77	1774.5	1780.5	1774.5	1782	2601.3	2627.5
56.25%		876.28	872.36	1768.7	1755.5	1768.7	1768.4	2600.8	2622
6.25%	0/90	887.42	894.06	1819.7	1840	1819.7	1845	2489.8	2542.1
25%		887.42	885.73	1817.4	1824.6	1817.4	1825.8	2489.7	2501.2
56.25%		887.39	872.61	1810	1816.8	1810	1819.3	2489.1	2484.1

weakens and this further reduces the stiffness of the structure.

Effect of stacking sequence. In order to study the effect of stacking sequences on the natural frequencies of 6.25%, 25% and 56.25% of the delaminated area (8-layered, three types of stacking sequences (0/90/45/90), (0/90) and (0/45) are considered. The changes in the natural frequency corresponding to their stacking sequences are presented in Table 3.

From Table 1, for mode 1, it is observed that highest value of natural frequency is observed for (0/90) sequence with all sides clamped and highest second and third values are observed in all sides clamped boundary conditions for (0/90/45/90) and (0/45) respectively. This is valid for mode 2 and mode 3. For mode 4, (0/45) has the highest values of natural frequencies for all delamination sizes.

Table 3. Effect of stacking sequence for C-C-C-C boundary condition subjected to delamination sizes of 6.25%, 25% ans 56.25%

Delamination area	Stacking	Mode1 [Hz]	Mode2 [Hz]	Mode3 [Hz]	Mode4 [Hz]
	0/90/45/90	883.41	1803.8	1803.8	2538
6.25%	0/45	876.31	1776.4	1776.4	2601.4
	0/90	887.42	1819.7	1819.7	2489.8
	0/90/45/90	883.41	1801.7	1801.7	2538
25%	0/45	876.31	1774.5	1774.5	2601.3
	0/90	887.42	1817.4	1817.4	2489.7
	0/90/45/90	883.38	1795.2	1795.2	2537.4
56.25%	0/45	876.28	1768.7	1768.7	2600.8
	0/90	887.39	1810	1810	2489.1

Conclusion

In the present study, a single delamination at the middle of the plate is analyzed using three types of methods i.e analytical, experimental and finite element methods. The results are compared for all sides clamped boundary constraint. The findings of the influence of delamination size and stacking sequences, on the natural frequencies of carbon fibre reinforced composite plates are concluded as below.

- Analytical and finite element results have negligible differences.
- The natural frequencies decrease with an increase of delamination size.
- The natural frequencies not only depend on the delamination size but it also get influenced by the stacking sequences.
- The natural frequencies are less impacted in lower modes than higher modes in all stacking sequences.

Conflict of Interest. The authors declare no conflict of interest.

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